ADMM for Control of Mixed Traffic Flow with Human-driven and Autonomous Vehicles

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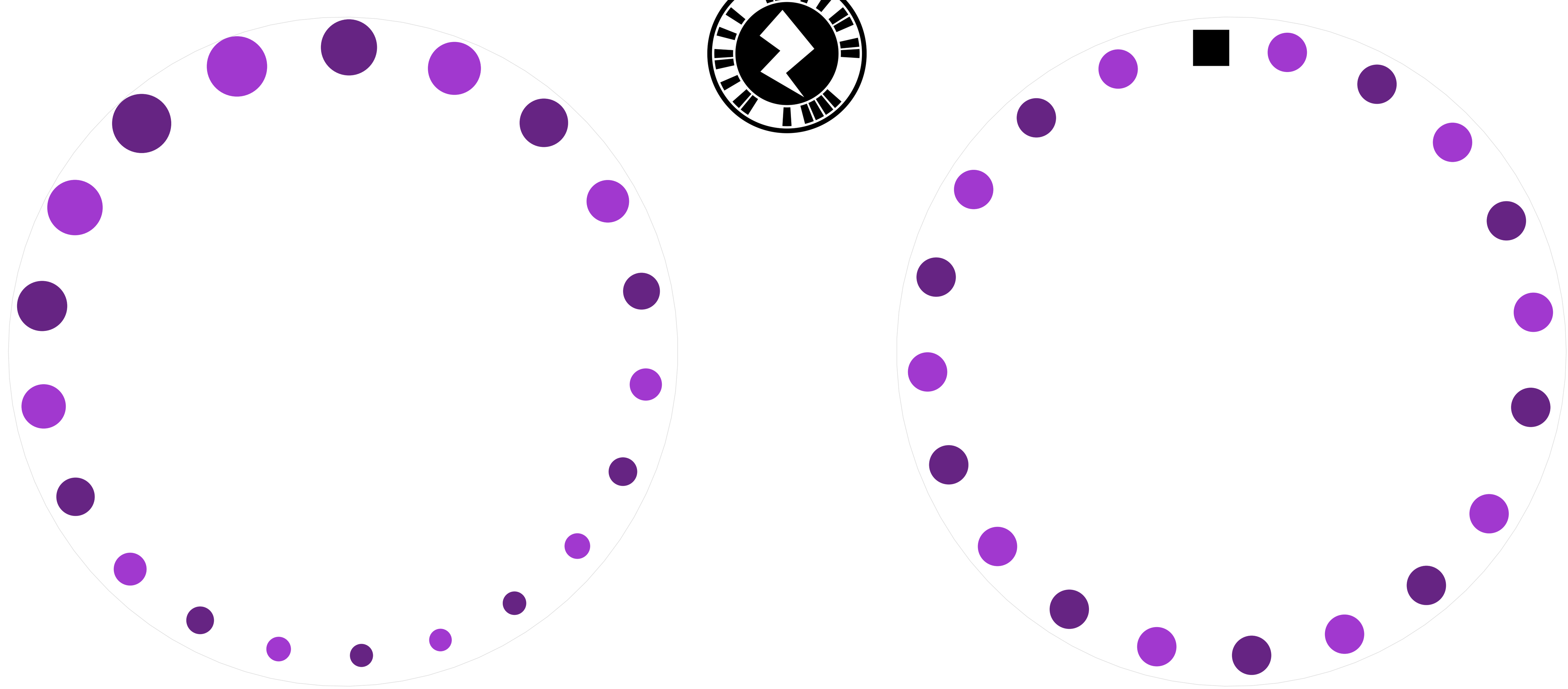

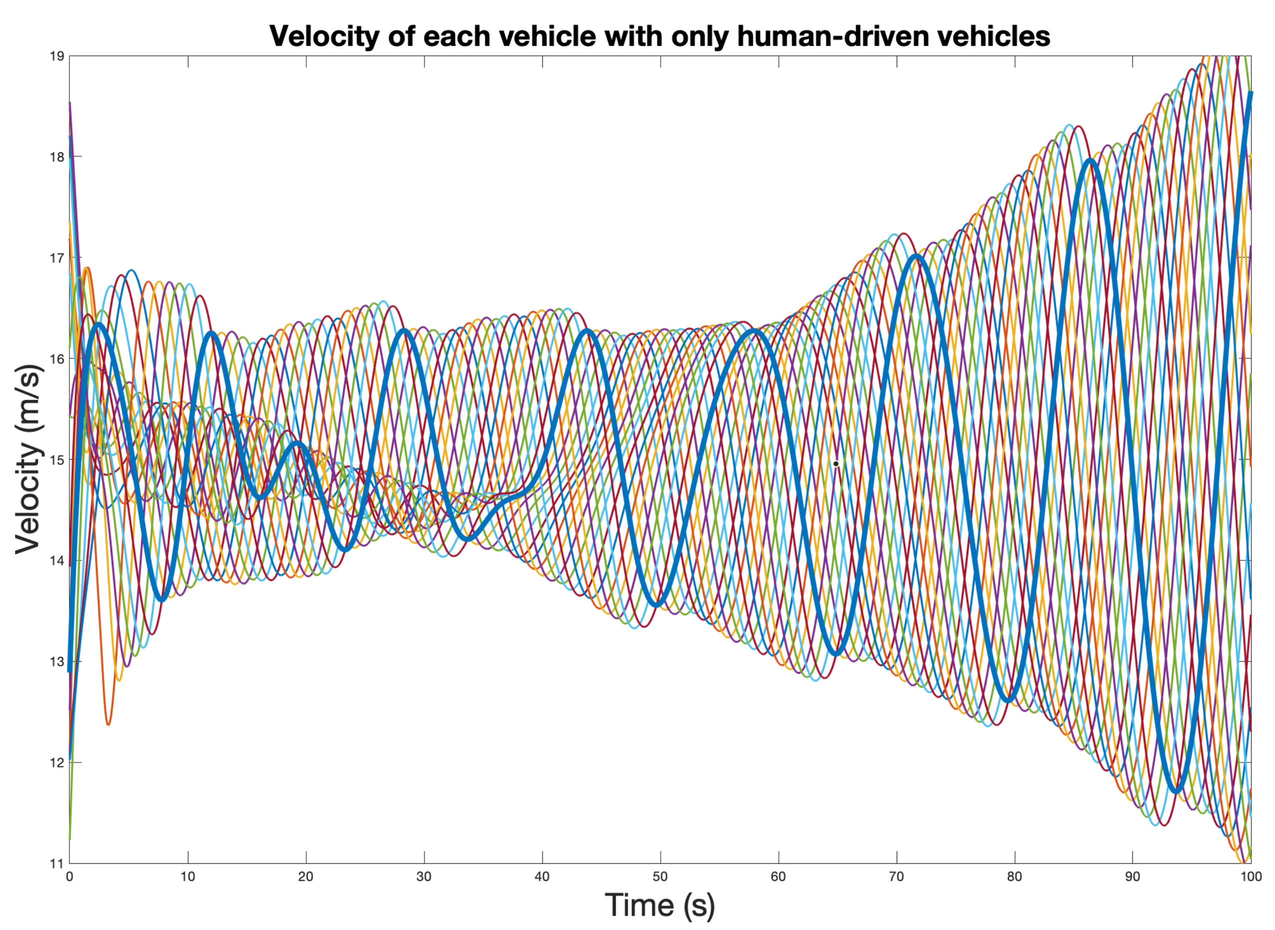

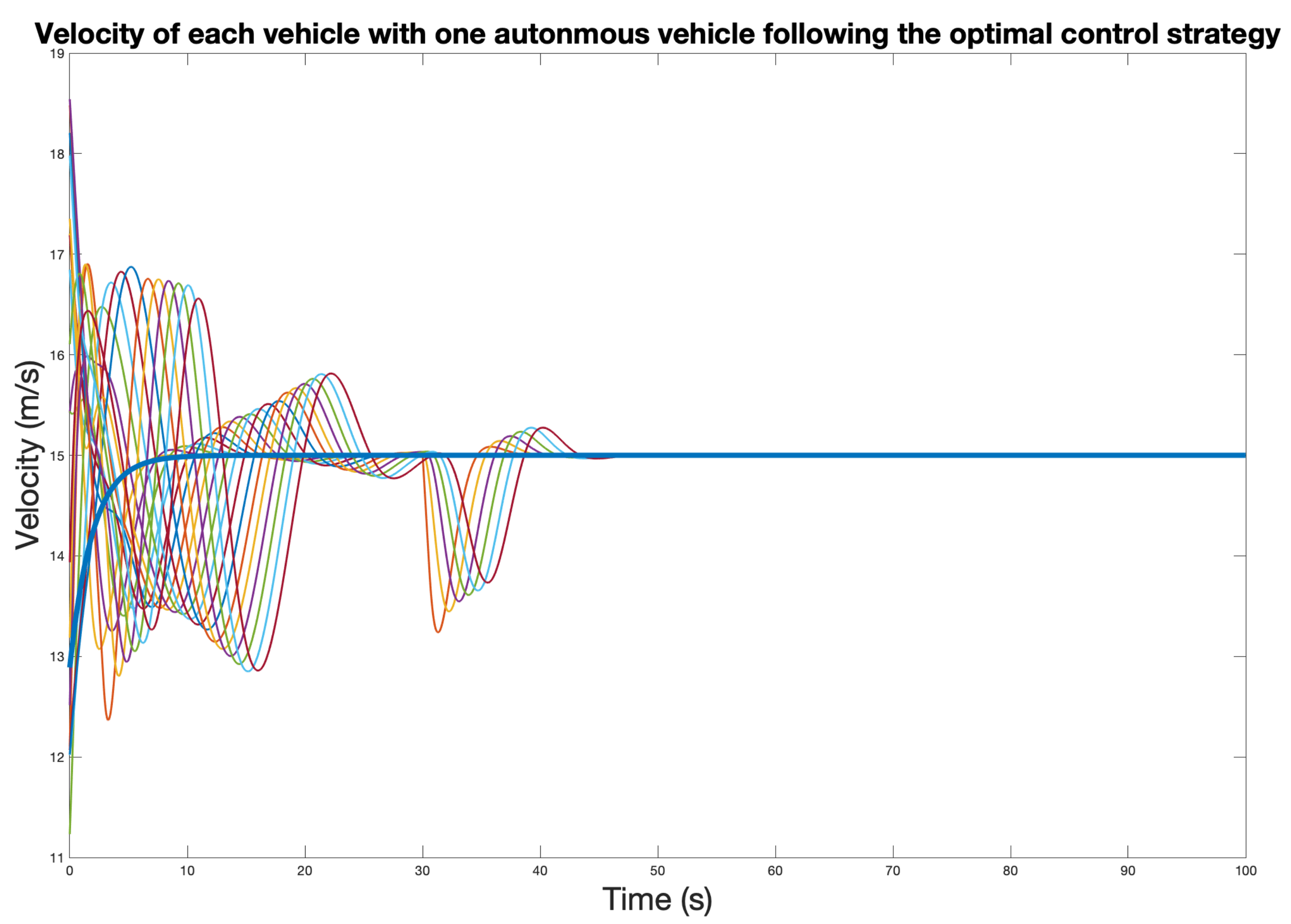

Problem Overview

- Traffic **congestion** leads to longer travel times and increased carbon emissions.
- True dynamics of congestion are unknown.
- Traffic flow is modelled as a **non-linear** dynamical system.
- **One** autonomous vehicle is added.
- An optimal control problem is created to minimise the impact of **disturbances**.
- Model **privacy** is conserved by solving in a **decentralised** way.
- Model privacy is needed to protect **data safety**.

Methods and Outcome

- Alternating direction method of multipliers (**ADMM**) was used to solve the optimal control problem.
- Ideas from **chordal decomposition** of sparse block matrices are used.
- The optimal control problem was converted to an **SDP**.
- The optimal control problem was solved using **centralized** control.
- ADMM algorithm was incorporated into the SDP.
- Further research will concentrate on finding ways to improve **convergence.**

Modelling the Traffic Flow Problem

The non-linear model is simplified into a standard linear dynamic model $\dot{x}(t) = Ax(t) + Bu(t) + H\omega(t)$, where

$$A = \begin{bmatrix} A_1 & 0 & \cdots & 0 & A_2 \\ A_2 & A_1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & A_2 & A_1 & 0 \\ 0 & \cdots & 0 & C_2 & C_1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, H = \begin{bmatrix} H_1 & 0 & \cdots & 0 \\ 0 & H_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & H_1 \end{bmatrix},$$

$$x_i = \begin{bmatrix} \tilde{s}_i \\ \tilde{v}_i \end{bmatrix} = \begin{bmatrix} s_i - s^* \\ v_i - v^* \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & -1 \\ \alpha_1 & \alpha_2 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ 0 & \alpha_3 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, C_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, H_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

- i is the **vehicle number**, where $i = 1 \dots N$.
- **Autonomous** vehicle is represented by $i = N$.
- s_i is the **distance** between vehicle i and $i-1$, s^* is the equilibrium spacing.
- v_i is the **velocity** of vehicle i, v^* is the equilibrium velocity.
- $u(t)$ is the **control** input, $\omega(t)$ is the **disturbance**.
- $\alpha_{1,2,3}$ represent the driver's **sensitivity** to errors in the spacing /velocity, they follow a uniform distribution.

Optimal control problem

The objective function minimizes the impact of the **disturbance** on the outputs of the system.

$$\min_K \|G_{z\omega}\|^2$$
subject to $u = -Kx$, $K \in \mathcal{K}$

Problem reformulated into an SDP with **linear** constraints instead of the $K \in \mathcal{K}$ sparsity constraint.

$$\min_{X,Y,Z} \mathrm{Trace}(QX) + \mathrm{Trace}(RY)$$
subject to $AX + XA^T - BZ - Z^TB^T + HH^T \preccurlyeq 0$,

$$\begin{bmatrix} Y & Z \\ Z^T & X \end{bmatrix} \succcurlyeq 0, X \succcurlyeq 0, ZX^{-1} \in \mathcal{K}$$

ADMM

First order **splitting method** for optimisation,

$$\min_{x,y} f(x) + g(y)$$
subject to $Ex + Fy = c$

$$x^{h+1} = \underset{x}{\mathrm{argmin}} \left(f(x) + \frac{1}{2}\rho \left\| Ex + Fy^h - c + \lambda^h \right\|^2 \right)$$
$$y^{h+1} = \underset{y}{\mathrm{argmin}} \left(g(y) + \frac{1}{2}\rho \left\| Ex^{h+1} + Fy - c + \lambda^h \right\|^2 \right)$$
$$\lambda^{h+1} = \lambda^h + Ex^{h+1} + Fy^{h+1} - c$$

where $\rho > 0$ is a **penalty parameter** and λ is the dual variable of the penalty parameter.