Tutorial 3: Dynamic Games

1. Formulating an extensive form game:
   Consider a 2 player game in which player 1 can choose A or B. The game ends if she chooses A, while it continues to player 2 if he chooses B. Player 2 can then choose C or D with the game ending if C is chosen, and continuing again to player 1 if D is chosen. Player 1 can then choose E or F, with the game ending either choice.
   (a) Model this as a game tree.
   (b) How many pure strategies does each player have?
   (c) Identity the subgames of this game.
   (d) Suppose that choice A gives utilities (2, 0) (i.e., 2 to player A, 0 to player E), choice C gives (3, 1), choice E gives (0, 0), and F gives (1, 2). Then what are the pure Nash equilibria of the game? What SPNE outcome(s) does Zermelo’s algorithm yield?

2. Imperfect information games:
   Consider an extensive form imperfect information game in which each player i has k information sets, that is, |I_i| = k for all 1 ≤ i ≤ n.
   (a) If a player has an identical number of m possible actions in each information set, how many pure strategies does he have?
   (b) If a player has m_j actions in the j’th information set (1 ≤ j ≤ k) how many pure strategies does he have?

3. Iterated games:
   Let’s consider playing the infinitely repeated prisoner’s dilemma, using finite state automata strategies, and measuring utility over infinite runs as the average utility obtained, as discussed in the lecture. Recall that the payoff matrix for the (one shot) prisoner’s dilemma is as follows:

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<thead>
<tr>
<th></th>
<th>defect</th>
<th>coop</th>
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</thead>
<tbody>
<tr>
<td>defect</td>
<td>-2</td>
<td>-3</td>
</tr>
<tr>
<td>coop</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
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   Figure 1 shows three two-state strategies for playing the iterated prisoner’s dilemma.
Figure 1: Three finite state machines that play the iterated prisoner’s dilemma.

(a) Informally explain what TAT-FOR-TIT does.

(b) Consider each strategy playing against each other strategy (including itself). Compute the runs that would be generated, and identify the finite but infinitely repeating sequence of outcomes. Use this repeating sequence to compute the utility obtained by each strategy in each pairing.

(c) Which of these pairs of strategies do you think forms a Nash equilibrium? (An informal argument will suffice.)

4. Iterated games:

This question is for personal development only, and is not strictly speaking part of the assessment!

Consider all possible two-state finite machines for playing the iterated prisoner’s dilemma.
(a) Suppose one of these machines has both states labelled with the same action (we have either $C$ in both ovals, or $D$ in both ovals). What behaviour does it generate? Can you simplify the automaton at all?

(b) Let us say two automata strategies $\sigma_1$ and $\sigma_2$ are distinct if there is an automaton $\sigma^*$ such that the sequence of outcomes generated by playing $\sigma^*$ against $\sigma_1$ is different to the sequence of states generated by playing $\sigma^*$ against $\sigma_2$.

Claim: there are precisely 26 distinct one and two state automata. (The one-state automata are $A \ L \ L \ D$ and $A \ L \ L \ C$.)

Of these 26 automata strategies, 13 will start by playing $C$, and the other 13 start by playing $D$. Draw the 13 automata that start by playing $C$.

Given these 13 automata, you can very easily obtain the 13 automata that start by playing $D$. So do it.

5. Minimax theorem in zero sum games (pure strategies).

Prove the minimax theorem for pure strategies, as stated in the lecture.