Unconventional Positioning Technology

Andrew Markham

Department of Computer Science, University of Oxford
Overview

- Unconventional Positioning Technologies
- Magnetic Fields
  - Geomagnetic
  - Generated
  - Applications
Section 1

Unconventional Positioning Technologies
Traditional positioning techniques

- Radio based position is the *de facto* solution for the majority of positioning applications
  - GPS: Revolutionized the world
  - Cell Tower/GSM: High density
  - WiFi/BLE: Indoor positioning

- However, there are some applications where the dominant solutions perform poorly or simply fail:
  - Underground
  - Underwater
  - Indoors
LEDs can be modulated to uniquely act as landmarks\textsuperscript{1}

\textsuperscript{1}http://wise.ece.cmu.edu/redmine/projects/vlc/wiki
A room has a unique acoustic transfer function. This can be used like a fingerprint.²

²Indoor localization without infrastructure using the acoustic background spectrum, Mobisys ’11
Geolocators are tiny (0.5g) devices that position migrating animals twice a day.\textsuperscript{3}

Section 2

Positioning using the Geomagnetic field
Positioning using the Geomagnetic field

The natural geo-magnetic field

- People have been using the Earth’s magnetic field for navigation

- Animals too! They can sense the geomagnetic field (magnetoception)
Earth’s magnetic field can be well approximated by a magnetic dipole.

- The intensity of the magnetic field varies from 0.25 (equator) to 0.65 (poles) Gauss.
Positioning using the Geomagnetic field

Natural Magnetic Fields

- Intensity of Earth’s Magnetic Field
Positioning using the Geomagnetic field
Natural Magnetic Fields

- The angle of the field also alters over the surface of the Earth
- Submarines in particular have exploited these microvariations to position themselves without suffering from Gyro drift
- Accuracy is relatively coarse (kilometres)
Positioning using the Geomagnetic field
Distortions of the Earth’s Magnetic Field

- Indoors, ferromagnetic material, especially in reinforced concrete, distorts the Earth’s magnetic field
A spatial map of these distortions can be built, using techniques very similar to HORUS.

The distortions themselves are not spatially unique.

However, for someone moving through the area, the sequence of distortions can be exploited to provide positioning.

Typical accuracy is around 2-3 m and various startups (IndoorAtlas) are exploiting this technique.
Positioning using the Geomagnetic field

Distortions of the Earth's Magnetic Field

Advantages:

- No infrastructure needs to be deployed
- Virtually all smart-phones have a magnetometer for controlling screen rotation
- Adds absolute location to traditional IMU
Disadvantages:

- Map has to be built and maintained
- Users can only be tracked when they move
- Accuracy is not good enough for many applications
Section 3

Generating our own magnetic fields
Generating Artificial Magnetic Fields

- We can produce magnetic fields using coils of wire
  - \( B \propto INA \)
  - \( I = \) current, \( N = \) number of turns, \( A = \) cross-sectional area
- We can also sense a time-varying field using a coil of wire
  - \( V \propto B \cos(\theta) \)
Modulation

- Modulate the signal to make it easier to detect, especially compared with the strong Earth’s magnetic field
Magnetic fields fall off more rapidly

\[
\text{RSSI} \propto \frac{1}{r^2} \quad \text{40 dB/decade}
\]

\[
|B| \propto \frac{1}{r^3} \quad \text{60 dB/decade}
\]
RSSI vs MI: Decay

- Magnetic fields fall off more rapidly
- Range less than traditional radio

\[
\begin{align*}
\text{RSSI} & \propto \frac{1}{r^2} \\
40 \text{ dB/decade} & \\
|B| & \propto \frac{1}{r^3} \\
60 \text{ dB/decade} &
\end{align*}
\]
RSSI vs MI: Antennas

- Magnetic field controlled by size and shape of generating coil

![RSSI and MI diagrams](image-url)
Magnetic field controlled by size and shape of generating coil
Magnetic field controlled by size and shape of generating coil
Magnetic field controlled by size and shape of generating coil
RSSI vs MI: Antennas

- Magnetic field controlled by size and shape of generating coil
- Simple to alter field patterns to optimize localization

![RSSI and MI Diagrams]
MI penetrates any non-metallic objects and does not suffer from multipath.
MI penetrates any non-metallic objects and does not suffer from multipath

Environmental obstacles do not affect MI localization accuracy
MI is a *vector* field i.e. it has magnitude and direction.
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Using a triaxial sensor, we can make the device rotationally invariant
Magnetic Fields for Navigation
Low-Frequency Magnetic Fields

- We can electronically control the orientation and the magnitude of the magnetic moment by using 3 mutually perpendicular coils.

![Triaxial coils](image)

*Figure*: Triaxial magnetic transmitter (TX) and receiver (RX) operating at 2.5 kHz.

- The sensor is also equipped with a triaxial coil.
- The total received power is invariant to TX and RX orientation.
Typical Measurements
Overall RSSI

- The abrupt decay of the magnetic field dB-magnitude corresponds to a path-loss exponent of 6
  - limits the transmission range
  - enables easy detection of tiny changes in distance (few cm)

![Estimated channel model TX1](image1.png)

![Estimated channel model TX2](image2.png)

Figure: The overall magnetic RSSI measured outdoor decays at 60 dB/decade.
Section 4

Applications of MI positioning
Above-ground tracking

- Tracking animals above ground is relatively well researched
  - ZebraNet - GPS-WSN
  - Virtual Fencing - GPS-WSN
  - TurtleNet - GPS-WSN
  - WildSensing - RFID-WSN
  - ...

What about burrowing animals?

Example burrowing species: badgers

- Nocturnal medium sized carnivores (8 kg)
- Live in extensive (20 m x 10 m) setts
- Tunnels between 1 and 3 m deep typically
- 1 to 20 badgers in a sett
Animal Tag: MI Sensor

- MI sensor detects signals from three orthogonal transponders
- Simultaneously measures RSSI
- Vector magnitude taken to ensure rotational invariance

$$|B| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$
Antennas
Positioning

- Using signal strengths from 4 or more antennas, we can work out likely location of animal
- This is done using a simple particle filter approach
- Not limited to 2-D, but can also be used in 3-D
Results

- 20 badgers tagged, each generates about 1 million readings per day
- Typical 3-D positioning accuracy of 30 cm RMS\textsuperscript{4}

\textsuperscript{4}Underground Localization in 3D using magneto-inductive tracking, IEEE Sensors 2012
2-D Accuracy
Summary

- Using MI tracking solved a previously “impossible” problem
- Leading to novel insights about animal behaviour
- Latest generation also have accelerometer, so we can also mine activity
Section 5

Precise 3-D localization in Challenging Environments
3-D Magnetic Vector Modulation

- As previously mentioned we can:
  - Generate and steer a magnetic vector using 3 orthogonal transmitting coils
  - Sense and recover the magnetic vector using 3 orthogonal receiving coils
- This provides the *unique* ability to localize a receiver using a single transmitter
With 3 transmitting antennas and 3 receiving antennas, we have a total of 9 measurements, of which 6 are unique.

Using some physics\(^5\) we can work out the 6 degrees of freedom in the receiver (position and orientation).

Due to symmetry, we have a hemispherical ambiguity, which can be solved with some prior knowledge.

\(^{5, \text{in the appendix}}\)
Applications: Worker Safety

- Workers underground cannot currently be localized - major safety issue for railway and mining industries\(^6\)

\(^6\) TrackSafe Project
Sensor and Actuator Networks (CDT) Unconventional Positioning Technology

Applications: Structural Deformations

- Embed tiny triaxial magnetic sensors in concrete structures to measure deformation\(^7\)

\(^7\)www.mi6sense.org
Results: Outdoor tracking

- Walking in a circle around the transmitter
Results: Indoor tracking

- Operating through solid concrete over two floors

![Diagram showing indoor tracking through concrete floors with position error distribution](image)
Results: Precision tracking

- Precision (cm-level) ranging at a distance of 3 m.
Section 6

Conclusions
If some property varies over space, then there is a strong chance that someone has or will use it for positioning!

Magneto-Inductive positioning has some unique properties which make it an interesting alternative

Positioning itself though is just another sensor - it is what we do with it that is important
Section 7

Appendix
Consider TX located at the origin \((x, y, z) = (0, 0, 0)\)

Let RX position in 3D be described by the position vector 
\[
r = (x_r, y_r, z_r)
\]

The range is 
\[
r = \|r\|_2 = \sqrt{x_r^2 + y_r^2 + z_r^2}
\]

TX is energized in each axis, and the corresponding magnetic
moments are:

\[
m_i = N_{TX} I_{TX} A_{TX} e_i,
\]

where

- \(N_{TX}\) = number of turns of the TX coil
- \(I_{TX}\) = the coil input TX coil current
- \(A_{TX}\) = the area of the TX coil
- \(e_i, \ i = 1, 2, 3\) are the standard Euclidean basis vectors (excitations)
**GOAL:** estimate the 3D position of RX

The \( B \)-field at an arbitrary position \( r \), given an arbitrary magnetic moment \( m \) is:

\[
B(r, m) = \frac{\mu_{TX}}{4\pi} \left[ \frac{3r(m^T r)}{r^5} - \frac{m}{r^3} \right]
\]

\[
= \frac{\mu_{TX}}{4\pi r^3} \left[ \frac{3rr^T}{r^2} - I_3 \right] m,
\]

where

- \( \mu_{TX} \) is the magnetic permeability of the TX coil core
- \( I_3 \) denotes the \( 3 \times 3 \) identity matrix
- \((\cdot)^T\) denotes the matrix transpose

For each TX magnetic moment \( m_i, i = 1, 2, 3 \), we get a vector \( b_i = B(r, m_i) \)
Define the matrix whose columns are $b_i, \ i = 1, 2, 3$

$$B_{1,2,3} \triangleq [b_1, b_2, b_3] = \frac{\mu_{TX} N_{TX} I_{TX} A_{TX}}{4\pi r^3} \left[ \frac{3rr^T}{r^2} - I_3 \right] [e_1, e_2, e_3] \quad (3)$$

Let $\Omega \in SO(3)$ be an orthogonal matrix describing the orientation of the RX frame w.r.t. the TX frame

Then, the magnetic vector field described in the RX frame is

$$\Omega B_{1,2,3} = \frac{\mu_{TX} N_{TX} I_{TX} A_{TX}}{4\pi r^3} \Omega \left[ \frac{3rr^T}{r^2} - I_3 \right] \quad (4)$$
The voltages induced in the RX \((x, y, z)\)-axes due to TX excitations \((e_1, e_2, e_3)\) are described by the following channel matrix:

\[
S \triangleq \begin{bmatrix}
S_{x1} & S_{x2} & S_{x3} \\
S_{y1} & S_{y2} & S_{y3} \\
S_{z1} & S_{z2} & S_{z3}
\end{bmatrix} = 2\pi f \mu_{RX} N_{RX} A_{RX} \Omega B_{1,2,3}
\]  

where

- \(f\) is the frequency of the excitation
- \(\mu_{RX}\) is the magnetic permeability of the RX coil core
- \(N_{RX}\) = number of turns of the RX coil
- \(A_{RX}\) = the area of the RX coil
Define the range-dependent scaling factor that also incorporates all the TX/RX coils specific constants

\[
c = \frac{1}{r^3} 0.5 f \mu_{TX} \mu_{RX} N_{TX} N_{RX} I_{TX} A_{TX} A_{RX}
\] (6)

Then, we can write:

\[
S = c \Omega \left[ \frac{3rr^T}{r^2} - I_3 \right].
\] (7)

Note that the channel matrix depends on the position vector \( r \) we are interested in, and the orientation \( \Omega \).
The channel matrix $S$ containing the position information can be estimated at the RX using a known transmitted preamble.

The input-output relationship of the $3 \times 3$ channel is

$$P_{RX} = P_{TX}S^T + \text{(Gaussian noise)},$$

where $P_{TX}$ and $P_{RX}$ are the $N \times 3$ transmitted and received preamble, respectively (tall matrices).

Therefore, the LS estimate (which in this case is also Maximum Likelihood estimate) of the channel matrix is

$$\hat{S} = [P_{TX}^\dagger P_{RX}]^T,$$

where $(\cdot)^\dagger$ denotes the Moore-Penrose pseudoinverse.
Let us analyze a bit the channel matrix and try to infer the range $r$:

$$S = c\Omega \left[ \frac{3rr^T}{r^2} - I_3 \right], \quad (10)$$

The scaling factor $c \propto r^{-3}$, and therefore contains the range information.

The latter factor depends only on the versor $r/\|r\|$ (i.e., only on the direction of the position vector, not on its magnitude).

Therefore, in free-space, the Frobenius norm of $S$ also decays with the cube of the range, and can be used for range estimation:

$$\|S\|_F \propto r^{-3}, \quad (11)$$

Since orthogonal matrices preserve the Frobenius norm, the range estimate is invariant w.r.t. TX/RX relative orientation.
Define the overall RSSI (Received Signal Strength Indicator) measured in dB as

\[ \rho = 20 \log \| S \|_F \]  

(12)

Since \( \| S \|_F \propto r^{-3} \), the law describing the RSSI vs. distance in free-space is

\[ \rho = \rho_0 - 60 \log(r/r_0), \]

(13)

where \( \rho_0 \) is the RSSI measured at some reference distance \( r_0 \)

Therefore, the range estimate in free-space is

\[ r = r_0 10^{(\rho_0 - \rho)/60} \]  

(14)

When plotting the RSSI vs the log-distance \( \log(r/r_0) \), the slope of the line is 60dB/decade
Our next goal is to determine the position vector $\mathbf{r}$

Its modulus (the range) $r = \|\mathbf{r}\|$ is known by now, we only need to determine its direction in 3D.

Recall that

$$\mathbf{S} = c\mathbf{\Omega} \left[ \frac{3\mathbf{rr}^T}{r^2} - \mathbf{I}_3 \right],$$

(15)

We first get rid of the arbitrary RX orientation $\mathbf{\Omega} \in SO(3)$ as follows

Define the channel inner product matrix

$$\mathbf{C} = \mathbf{S}^T \mathbf{S}$$

(16)

From Eqs. (15) and (16), obtain

$$\mathbf{C} = c^2 \left[ \frac{3\mathbf{rr}^T}{r^2} - \mathbf{I}_3 \right]^T \mathbf{\Omega}^T \mathbf{\Omega} \left[ \frac{3\mathbf{rr}^T}{r^2} - \mathbf{I}_3 \right] = c^2 \left[ 3 \frac{\mathbf{r}}{\|\mathbf{r}\|} \frac{\mathbf{r}^T}{\|\mathbf{r}\|} - \mathbf{I}_3 \right]^2$$

(17)
Let $C = UDU^T$ be the eigendecomposition of $C$

We get

$$\frac{r}{\|r\|} \frac{r^T}{\|r\|} = \frac{1}{3c} C^{1/2} U D^{1/2} U^T + \frac{1}{3} I_3 = U \left[ \frac{1}{3c} D^{1/2} + \frac{1}{3} I_3 \right] U^T$$

which is a rank-one matrix

Consequently, the maximal eigenvector $u_{\text{max}}$ of $C$ is the position versor we are interested in:

$$\frac{r}{\|r\|} = u_{\text{max}}$$

Finally the 3D position vector can be written as

$$r = ru_{\text{max}}$$