Outline of this module

• Instructors:
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• Teaching Assistants:
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• Schedule for this week:
  Lectures (Mon-Fri) 9:30-12:00
  Exercises on theory/applications (Mon-Thu) 14:00-17:00

• Assessment:
  exercise attendance and work on case study running through the week and supported by TA
Outline of this module

- **Monday**: general introduction, modelling, I/O models (frequency vs. state space), linearisation, simulations in MATLAB, hints at Simulink

- **Tuesday**: LTI models, analytic solutions, dynamical stability, controllability and observability

- **Wednesday**: optimisation in control; LMIs, duality theory, conic programming, Lyapunov via SOS

- **Thursday**: control synthesis; state feedback via pole placement and via LQR, output feedback (filters), notes on MPC

- **Friday**: vistas in Control Theory
Textbooks

• Hespanha, Linear Systems Theory

• Aström and Murray, Feedback Systems – online

• Boyd and Vandenberghe, Convex Optimization – online
A short survey

what is your existing background in systems and control theory?
A short history of Control Theory

• pre-1940s: control in frequency

• 1940s: Wiener’s cybernetics, filtering, applications

• 50s: Ragazzini and Zadeh, state-space

• 60s: Kalman filtering, linear systems theory, rocket science

• 70s: stochastic control

• 80s: adaptive and robust control

• 90s: non-linear, geometric control
• 2000s: optimisation in control, MPC, hybrid systems, networks

• nowadays: inter-disciplinary research, applications (biology, internet, finance, . . . )
Lecture 1

- The concept of “feedback”
- The concept of “model” in systems engineering: state-space models
- The concept of “controller”: controlling a model via feedback
The concept of feedback

- Compare the following two interconnections:

  ![Diagram showing two interconnections:](image)

  **series connection**
  **(open loop)**

  **feedback connection**
  **(closed loop)**

- dynamical feedback, control feedback
The concept of feedback – Watt’s Regulator

- centrifugal governor (flyball governor) for steam engine
The concept of feedback – TCP Protocols

global network

local network
The concept of feedback – Eucariotic Cell

Figure 1.12: The wiring diagram of the growth-signaling circuitry of the mammalian cell [HW00]. The major pathways that are thought to play a role in cancer are indicated in the diagram. Lines represent interactions between genes and proteins in the cell. Lines ending in arrowheads indicate activation of the given gene or pathway; lines ending in a T-shaped head indicate repression. (Used with permission of Elsevier Ltd. and the authors.)

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The concept of feedback – Predator/Prey Ecosystems
The use of formal models

• Objective: abstraction from real system, quantitative description of its underlying dynamics

• How to build a model?
  – physics-based model (conservation laws, physical geometry)
  – models based on known interactions and properties (e.g.: energy-based models, stoichiometric models)
  – models from experiments (data driven): measurement of model properties, model building via fitting, use of transfer functions

• use of black box, vs grey box models
State-space Models: a First Example

- mass-spring-damper system

\[ F = m\ddot{q}(t) \]
\[ \ddot{q}(t) = \frac{1}{m} \left( -c(\dot{q}(t)) - kq(t) + u(t) \right) \]
State-space Models: a First Example

\[ \ddot{q}(t) = \frac{1}{m} \left( -c(\dot{q}(t)) - kq(t) + u(t) \right) \]

- State: \( q(t) \)
- Input signal: \( u(t) \)
- Output signal: \( y(t) = q(t) \)
State-space Models: a First Example

\[ \ddot{q}(t) = \frac{1}{m} (-c(\dot{q}(t)) - kq(t) + u(t)) \]

- Block diagram for input-output relationship

- Introduce state variables (integrator outputs):
  \( x_1(t) = q(t) \) and \( x_2(t) = \dot{q}(t) \)
• Obtain system of first-order ODE:

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= \frac{1}{m} (-c(x_2(t)) - kx_1(t) + u(t))
\end{align*}
\]

• To find solution, need two initial conditions

• Note presence of linear \((kx_1)\) & nonlinear parts \((c(x_2))\)

• Given a model, we can
  – analyse it (formal proof, verification)
  – simulate it (testing, validation)
  – control it (synthesis)
State-space Models: a First Example

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= \frac{1}{m} (-c(x_2(t)) - kx_1(t) + u(t))
\end{align*}
\]
State-space Models: a Second Example

- Predator-prey dynamics in closed ecosystem (introduced before)

- State variables:
  - time-dependent population level for the lynxes: \( l(t), t \geq 0 \)
  - and for the hares: \( h(t), t \geq 0 \)

- Control Input: hare birth rate \( b(u) \), function of food

- Outputs: population levels \( l(t), h(t) \)

- Model parameters:
  - Mortality rate \( d \). Interaction rates \( a, c \)
• Model as abstraction of population dynamics:

\[
\begin{align*}
\dot{h}(t) &= b(u)h(t) - a l(t)h(t) \\
\dot{l}(t) &= c l(t)h(t) - d l(t)
\end{align*}
\]

• Simulation outputs of developed model:
State-space Models: a Third Example

- Control of inverted pendulum on moving cart
  (in modern terms, a balance system, e.g. Segway)
State-space Models: a Third Example

• Dynamics can be derived via Lagrange equations

• States: position $p$ and angle $\theta$

• Kinetic energy:

$$T_M = \frac{1}{2} M \dot{p}^2, \quad T_m = \frac{1}{2} m (\dot{p}^2 + -2l \dot{p} \dot{\theta} \cos \theta + l^2 \dot{\theta}^2)$$

• Potential energy:

$$V = mgl \cos \theta$$

• (details discussed in Exercise session)
System Identification

- data-driven modelling, models from experiments


\[ u(1), u(2), \ldots, u(N) \]

\[ y(t) = G(s)u(t) \]

\[ y(1), y(2), \ldots, y(N) \]
System Identification: a panacea?

- physical models
  - general (nonlinear models)
  - based on physical considerations, allows understanding of structure
    - parameters obtained via nonlinear optimization → local minima, time-consuming

- black-box models
  - often very effective
  - intuitive behavior (in particular for 1st & 2nd order models)
  - identification is fast
  - many control design methods for linear models
    - black-box → relation with physical system?
    - linear → not general, only valid locally