Exercise 8
Controller Design for the Boeing 747 (Case Study IV)

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INTRODUCTION
We finally combine the results from Exercise 7 to design a robust controller for the (longitudinal model of the) Boeing 747.

PROBLEMS
1. Recall the model for the Boeing 747, with the state-space model

\[
\dot{x} = \begin{pmatrix}
-0.003 & 0.039 & 0 & -0.322 \\
-0.065 & -0.319 & 7.747 & 0 \\
0.020 & -0.101 & -0.429 & 0 \\
0 & 0 & 1.000 & 0
\end{pmatrix}
+ \begin{pmatrix}
0.01 & 1.00 \\
-0.18 & -0.04 \\
-1.16 & 0.60 \\
0 & 0
\end{pmatrix}
\begin{pmatrix}
u
\end{pmatrix}.
\]

Design a linear state feedback controller for this system that satisfies all of the following design requirements:

a) The controller minimizes, to the extent allowed by the other design constraints, the cost function

\[
\int_0^\infty \|x(\tau)\|_2^2 + \|u(\tau)\|_2^2 \, d\tau
\]

for any initial state \(x_0\).

Hints: First solve the optimization problem using \texttt{care}. Then try to reproduce the result using LMIs.

b) All closed-loop eigenvalues have imaginary parts less than 2.5 in magnitude and lie in a conic sector with inner angle of \(2\theta\), where \(\theta = 30^\circ\).

Hints: In the lecture, the Kronecker product was used to state the condition

\[
L \otimes P + M \otimes (AP) + M^T \otimes (PA^T) \prec 0
\]
in a compact way. In MATLAB, the Kronecker product can be computed using `kron` and Yalmip is able to handle it.

c) The controller is robustly stable to variations of up to 10% in the magnitude of the value $A_{23} = 7.747$ that appears in the matrix $A$.

**Hints:** Consider two matrices $A_1$ and $A_2$ with $(A_1)_{23} = A_{23}$ and $(A_2)_{23} = \overline{A}_{23}$, where the interval $[A_{23}, \overline{A}_{23}]$ covers the uncertainty of the element $A_{23}$ in the system matrix $A$.

d) Plot the eigenvalues of the controlled system for the system matrix equal to $A$, $A_1$, and $A_2$, respectively. Compare them to the eigenvalues for the controlled system resulting from a).