CDT
Autonomous and Intelligent Machines & Systems

Introduction to Modern Control
MT 2014

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Outline of this module

• Instructors and Teaching Assistants:
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  (also organisation with A. Papachristodolou)

• Schedule for this week:
  Lectures and Exercises (theory/simulations) 9:30-12:00, and 14:00-16:00
  Wednesday: no lecture, attendance of workshop in Bristol

• Assessment:
  via code generated on case study running through the week and
  supported by TA
Outline of this module

• Monday:
  general introduction, modelling, I/O models (frequency vs. state space),
  linearisation, simulations in MATLAB, Simulink

• Tuesday:
  LTI models, analytic solutions, dynamical stability, controllability and
  observability

• Thursday:
  optimisation in control; LMIs, duality theory, conic programming,
  Lyapunov via SOS

• Friday:
  control synthesis; state feedback via pole placement and via LQR,
  output feedback (filters), notes on MPC
Textbooks

- Hespanha, Linear Systems Theory
- Aström and Murray, Feedback Systems – online
- Boyd and Vandenberghe, Convex Optimization – online
A short survey

what is your existing background in systems and control theory?
Lecture 1

• The concept of “feedback”

• The concept of “model” in systems engineering: state-space models

• The concept of “controller”: controlling a model via feedback
The concept of feedback

- Compare the following two interconnections:

  ![Series connection (open loop) vs. Feedback connection (closed loop)]

  - series connection (open loop)
  - feedback connection (closed loop)

- dynamical feedback, control feedback
The concept of feedback – Watt’s Regulator

- centrifugal governor (flyable governor) for steam engine
The concept of feedback – TCP Protocols

global network

local network
The concept of feedback – Eucariotic Cell

Figure 1.12: The wiring diagram of the growth-signaling circuitry of the mammalian cell [HW00]. The major pathways that are thought to play a role in cancer are indicated in the diagram. Lines represent interactions between genes and proteins in the cell. Lines ending in arrowheads indicate activation of the given gene or pathway; lines ending in a T-shaped head indicate repression. (Used with permission of Elsevier Ltd. and the authors.)

Feedback is the science of reverse (and eventually forward) engineering of biological control networks such as the one shown in Figure 1.12. There are a wide variety of biological phenomena that provide a rich source of examples of control, including gene regulation and signal transduction; hormonal, immunological and cardiovascular feedback mechanisms; muscular control and locomotion; active sensing, vision and proprioception; attention and consciousness; and population dynamics and epidemics. Each of these (and many more) provide opportunities to figure out what works, how it works, and what we can do to affect it.

One interesting feature of biological systems is the frequent use of positive feedback to shape the dynamics of the system. Positive feedback can be used to create switchlike behavior through autoregulation of a gene, and to create oscillations such as those present in the cell cycle, central pattern generators or circadian rhythm.

Ecosystems. In contrast to individual cells and organisms, emergent properties of aggregations and ecosystems inherently reflect selection mechanisms that act on multiple levels, and primarily on scales well below that of the system as a whole. Because ecosystems are complex, multiscale dynamical systems, they provide a broad range of new challenges for the modeling and analysis of feedback systems.

Recent experience in applying tools from control and dynamical systems to bacterial networks suggests that much of the complexity of these networks is due to the presence of multiple layers of feedback loops that provide robust functionality.
The concept of feedback – Predator/Prey Ecosystems
The use of formal models

• Objective: abstraction from real system, quantitative description of its underlying dynamics

• How to build a model?

  – physics-based model (conservation laws, physical geometry)

  – models based on known interactions and properties (e.g.: energy-based models, stoichiometric models)

  – models from experiments (data driven): measurement of model properties, model building via fitting, use of transfer functions

• use of black box, vs grey box models
State-space Models: a First Example

- mass-spring-damper system

\[ \begin{align*}
F &= m\ddot{q}(t) \\
\ddot{q}(t) &= \frac{1}{m} \left( -c(\dot{q}(t)) - kq(t) + u(t) \right)
\end{align*} \]
State-space Models: a First Example

\[ \ddot{q}(t) = \frac{1}{m} \left( -c(\dot{q}(t)) - kq(t) + u(t) \right) \]

- State: \( q(t) \)
- Input signal: \( u(t) \)
- Output signal: \( y(t) = q(t) \)
State-space Models: a First Example

\[ \ddot{q}(t) = \frac{1}{m} (-c(\dot{q}(t)) - kq(t) + u(t)) \]

- Block diagram for input-output relationship

- Introduce state variables (integrator outputs):
  \( x_1(t) = q(t) \) and \( x_2(t) = \dot{q}(t) \)
• Obtain system of first-order ODE:

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= \frac{1}{m} \left( -c(x_2(t)) - kx_1(t) + u(t) \right)
\end{align*}
\]

• To find solution, need two initial conditions

• Note presence of linear \((kx_1)\) & nonlinear parts \((c(x_2))\)

• Given a model, we can

  – analyse it (formal proof, verification)
  – simulate it (testing, validation)
  – control it (synthesis)
State-space Models: a First Example

\[
\begin{aligned}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= \frac{1}{m} (-c(x_2(t)) - kx_1(t) + u(t))
\end{aligned}
\]
State-space Models: a Second Example

• Predator-prey dynamics in closed ecosystem (introduced before)

• State variables:
  – time-dependent population level for the lynxes: \( l(t), t \geq 0 \)
  – and for the hares: \( h(t), t \geq 0 \)

• Control Input: hare birth rate \( b(u) \), function of food

• Outputs: population levels \( l(t), h(t) \)

• Model parameters:
  – Mortality rate \( d \). Interaction rates \( a, c \)
• Model as abstraction of population dynamics:

\[
\begin{align*}
\dot{h}(t) &= b(u)h(t) - a l(t)h(t) \\
\dot{l}(t) &= c l(t)h(t) - d l(t)
\end{align*}
\]

• Simulation outputs of developed model:

![Graphs showing time dependent trajectories and phase plane with no noise and noisy conditions.](image-url)
State-space Models: a Third Example

- Control of inverted pendulum on moving cart
  (in modern terms, a balance system, e.g. Segway)
State-space Models: a Third Example

• Dynamics can be derived via Lagrange equations

• States: position $p$ and angle $\theta$

• Kinetic energy:

$$T_M = \frac{1}{2}M\dot{p}^2, \quad T_m = \frac{1}{2}m(\dot{p}^2 + -2l\dot{p}\dot{\theta}\cos\theta + l^2\dot{\theta}^2)$$

• Potential energy:

$$V = mgl\cos\theta$$

• Overall state $q = (p, \theta)$, input $u = F$, output $y = (p, \theta)$
• Lagrangian:

\[ L = T(q, \dot{q}) - V(q) = (T_M + T_m) - V \]

• Lagrange’s Equations:

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \begin{bmatrix} F \\ 0 \end{bmatrix}
\]

• Obtain

\[
\begin{bmatrix} (M + m) & -ml \cos \theta \\ -ml \cos \theta & ml^2 \end{bmatrix} \begin{bmatrix} \ddot{p} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} ml \sin \theta \dot{\theta}^2 \\ -mgl \sin \theta \end{bmatrix} = \begin{bmatrix} F \\ 0 \end{bmatrix}
\]

• Can synthetically write the dynamics as:

\[ M(q)\ddot{q} + K(q, \dot{q}) = Bu \]
System Identification


\[ u(1), u(2), \ldots, u(N) \quad \downarrow \quad y(1), y(2), \ldots, y(N) \]

\[ y(t) = G(s)u(t) \]
The identification procedure

1. **Experiment design & data collection**
   - Data

2. **Should data be filtered?**
   - Yes: **Polish and present data**
   - No: **Choice of model structure**

3. **Choice of model structure**
   - Data not ok
   - Model structure not ok

4. **Fit model to data**
   - Data

5. **Validate the model**
   - Can the model be accepted?
      - Yes
      - No
System Identification

• physical models
  + general (nonlinear models)
  + based on physical considerations
    - parameters obtained via nonlinear optimization $\rightarrow$ local minima, time-consuming

• black-box models
  + often very effective
  + intuitive behavior (in particular for 1st & 2nd order models)
  + identification is fast
  + many control design methods for linear models
    - black-box $\rightarrow$ relation with physical system?
    - linear $\rightarrow$ not general, only valid locally