Intelligent Systems for Price Taking and Market Making

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• 75% of US equity trades involve an autonomous system.

• **Getting it wrong is fatal:**
  – In August 2012, Knight Capital deployed defective code in its trading algorithm, costing them $440m in under an hour.
Trades in financial markets occur when one agent, the *price taker*, chooses to buy/sell at the ask/bid prices of a liquidity provider, the *market maker*.

- **Price taker objective**: anticipate the market, buy ahead of rallies, sell ahead of drops.

- **Market maker objective**: offset buy orders with sell orders, earn the gap between bid and ask price with minimal risk.
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  → What bid and ask prices should you show, subject to the dual constraint of profit maximisation and risk minimisation?
Bayesian Price Taking: Framework

**Inputs**
- Technical Indicators *used by chartist traders*
- Sentiment Analysis *reflecting retail investors*
- Analyst Opinions *reflecting institutional investors*
- Options Market Metrics *reflecting hedge funds*

**Model**
- Automatic Relevance Determination (ARD) Gaussian Process Regression

**Outputs**
- ARD Ranking of Feature Relevance
- Stock Market Return Sensitivities & Forecasts

**Evaluation**
- Comparison to Student-t Significance Tests
- Comparison to Autoregressive and Kalman Filter Models

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- ARD-GP NRMSE: 0.9740
  ARD-GP Corr: +0.3213
- AR NRMSE: 0.9885
  AR Corr: +0.1950

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Bayesian Price Taking: Technical Indicators

• Technical Indicator Definitions

\[ \text{kd EMA}(t) = \left( \text{Price}(t) - \text{kd EMA}(t-1) \right) \times \frac{2}{k + 1} + \text{kd EMA}(t-1) \]

\[ \text{MACD}(t) = \log \frac{12 \text{d EMA}(t)}{26 \text{d EMA}(t)} \]

\[ \text{Divergence}(t) = \text{MACD}(t) - \text{MACD}'s \ 9 \text{d EMA}(t) \]

• Finance is game-theoretic:
beliefs of participants shape financial reality.

S&P500 Returns as a function of Lagged Returns (x-axis) and MACD (y-axis). MACD matters.
Bayesian Price Taking: Sentiment Analysis

- **Social media sentiment data:** Twitter and Stocktwits
  - Twitter data not useful for market prediction: too much noise
  - Stocktwits sentiment *negatively* correlated with next-day returns!

- **Warren Buffett (2008):** “Be greedy when others are fearful, fearful when others are greedy.”
Bayesian Price Taking: Price Space Curvature

• Are price movements truly independent of price level?
  – aka, do asset prices really behave like Geometric Brownian Motion?
Bayesian Price Taking: Price Space Curvature

- Are price movements truly independent of price level?
  - aka, do asset prices really behave like Geometric Brownian Motion?
  - Prices frequently halt at option strike levels (red lines), and gap quickly in between.

Alphabet (NASDAQ: GOOG)  
Microsoft (NASDAQ: MSFT)  
Honeywell (NYSE: HON)  
Infosys (NSE: INFY)
Bayesian Price Taking: Options Market Metrics

• **Directionality and Viscosity:** Measures of directional bias and variance at different price points, using options market data.

\[
\text{Directionality}(t) = \sum_{s \in S} \left( OI(s,t)_{\text{Call}} - OI(s,t)_{\text{Put}} \right) - \sum_{s \in S} \left( OI(s,t-1)_{\text{Call}} - OI(s,t-1)_{\text{Put}} \right)
\]

\[
\text{Viscosity}(t) = \sum_{s \in S} \left( e^{-\lambda|\text{price}(t)-s|} \times \log[\min(OI(s,t)_{\text{Call}}, OI(s,t)_{\text{Put}})] \right)
\]

• **The mapping of Price Space to Return Space is not lossless:** price levels matter.

*S&P500 Returns as a function of Directionality (x-axis) and Viscosity (y-axis).*
Bayesian Market Making: Framework

- **Risk** dictates the optimal bid and ask prices for the market maker.

### Inventory Risk
- Asset-specific
- Well researched in finance literature (Avellaneda & Stoikov, 2008)
- Suited to a Control approach

### Adverse Selection
- Client-specific
- Under-researched
- Suited to a Learning approach
Bayesian Market Making: Inventory Risk

• **Inventory Risk**: balance profit maximisation with inventory risk aversion.
  – Wide bid-offer spread $\rightarrow$ no inventory risk, but also no profit potential
  – Tight bid-offer spread $\rightarrow$ capture the spread often, but violate risk constraints

• **Idea**: dynamically adjust bid and offer prices separately to encourage whichever side gets inventory to zero.

• **Stochastic Control solution**: dynamic programming solution drawing on Hamilton-Jacobi-Bellman equation.
Bayesian Market Making: Adverse Selection

- **Adverse Selection**: price taker exploits information advantage.

- **Idea**: learn the behaviour of each counterparty and adjust bid and offer prices punitively for adverse selectors.

- **Machine Learning solution**: ARD Gaussian Process representation of client behaviour to measure entropy as a proxy for adverse selection.
Conclusion

- **Autonomous Price Taking?**
  - Assess data relevance and retain only the salient features.
  - Learn multivariate mean surface of market returns.

- **Autonomous Market Making?**
  - Manage inventory risk as a feedback control problem.
  - Manage adverse selection by producing high-dimensional representations of counterparties, and tiering by entropy.
Appendix A: Do algorithms exacerbate volatility?

- Extreme events have rarified: Of the top 10 biggest single day DJIA declines in history, only 1 occurred in the last 2 decades (-7.9%, October 2008).

- Percentage daily returns on DJIA: Recent volatility not so different from a century ago.
Appendix B: Stochastic Control Solution (Avellaneda & Stoikov, 2008)

1. Market Maker Utility Function, \( v \):
\[
v(x, s, q, t) = E_t[-\exp(-\gamma(x + qS_T))]
\]

2. Optimal Policy Value Function, \( u \):
\[
u(x, s, q, t) = \max_{\delta^b, \delta^a} E_t[-\exp(-\gamma(x + qS_T))]
\]

3. Invoke Bellman Principle of Optimality:
\[
\max_{\delta^a, \delta^b} du = 0
\]
\[
u(s, x, q, T) = -\exp(-\gamma(x + qs))
\]

4. Write out the PDE implied by Bellman Eq.:
\[
\begin{align*}
&u_t + \frac{1}{2} \sigma^2 u_{ss} + \max_{\delta^b} \lambda^b(\delta^b) [u(s, x - s + \delta^b, q + 1, t) - u(s, x, q, t)] + \max_{\delta^a} \lambda^a(\delta^a) [u(s, x + s + \delta^a, q - 1, t) - u(s, x, q, t)] = 0 \\
u(s, x, q, T) &= -\exp(-\gamma(x + qs))
\end{align*}
\]